55. MASTERS OF SCIENCE IN PURE MATHEMATICS

1. INTRODUCTION

The Master of Science (M.Sc.) program in Pure Mathematics is intended to produce graduates whose areas of specialization are broad enough to carry out research, participate in national and international endeavors and disseminate knowledge through teaching.

We propose establishing a new Postgraduate Master's program in mathematics called Pure Mathematics within the school of Mathematics and Actuarial Sciences at Bondo University College(BUC)

. The Pure Mathematics program will offer an integrated interdisciplinary curriculum combining mathematics, and communication skills. The Pure mathematics two year Master's Program will be based upon a core curriculum and train students by having them participate in projects with practical deliverables. The program will place special emphasis on applications in science, engineering, and business.

Projects are selected by an industrial mathematics course advisory board.

The graduate will have studied not only the standard mathematical and statistical tools, but also the basic ideas of engineering and business, and will have received training in project development and in modes of industrial communication.

2. Rationale for the Program

The importance of computer science in science, engineering, and business requires no justification. On the other hand, the role of mathematics is arguably as important, but not nearly as well understood or appreciated. Broadly speaking as the complexity, difficulty, size, or structure of a problem grows, mathematical analysis assumes increasing importance. An interdisciplinary program combining mathematics and information sciences is an research area that is critical to science and technology and in which BUC can build a research program of national significance.

Our two year Pure Mathematics Master's Program will be structured so that graduates will have developed three overlapping skill sets:

1) their knowledge of *mathematics* will allow them to contribute to the solutions of complex problems requiring sophisticated analysis;

2) their knowledge of *computer science* will allow them to develop algorithms and software so that the solutions can be realized in practice; and

3) their facility with oral and written *communication skills* and project management will allow them to insert new technology into an organization. For this reason, our program will be interdisciplinary and place equal emphasis on mathematics, information sciences, oral and written communication skills, and project management.

Our approach is to focus on core courses together with a project-oriented curriculum so that the students leaving the two year Master's Program will have worked as a team member on one or more projects with practical deliverables. We expect that the majority of Master's students will take jobs in industry and that this particular combination of disciplines combined with a practical project orientation will provide them with a significant advantage when looking for jobs. We use an industrial mathematics advisory board to ensure that the projects we select and the students we graduate are of interest to industry.

Employment Opportunities:

Opportunities include systems engineer, analyst, research associate. Employment areas include federal and state governments, insurance companies, financial industry, as well as banks, automotives, aeronautics, and industrial companies.

2. OBJECTIVES

To produce graduates:

(i) With a wide background knowledge and basis techniques in Pure Mathematics

(ii) Who are able to carry out research and undertake Ph.D. research in their respective areas of specialization.

3. ADMISSION REQUIREMENTS

In addition to fulfilling the common university regulations, an applicant for the degree of Master of Science in Pure Mathematics should satisfy the following requirements.

(a) Hold a Bachelor's degree at least at lower second class honours from a university recognized By Bondo University College

(b) Must have taken Mathematics as a major subject in the first degree.

4. SUBMISSION AND PROCESSING OF APPLICATIONS FOR REGISTRATION

The common procedures approved by the senate shall apply.

5. DURATION OF THE PROGRAMME

The programme shall normally take a minimum of one (1) year and a maximum of four (4) years.

6. PROGRAMME REQUIREMENTS

The programme requires a minimum of thirty nine (39) units of coursework including thesis.

A student may take extra courses over and above the required number of units. A student may also choose courses subject to the approval of the department.

7. EVALUATION

(a) COURSEWORK AND EXAMINATION

(b) Each course shall be examined by a 2-hour end of semester written examination. This will account for 60% of the total mark in each course.

(c) Each course shall be examined by continuous coursework assessment comprising seminar papers, projects, reports, formal tests and participation in learning activities. This will account for 40% of the total mark in each course.

(d) The pass mark in each course (continuous assessment and written examination) shall be 50%.

(e) A candidate who fails half OR more than half of the units taken shall be discontinued.

(f) A candidate who fails less than half of the units taken shall sit for a supplementary examination.

(g) Each supplementary examination shall be awarded a maximum of 50%.

(h) A candidate who fails any paper taken as a supplementary examination shall be discontinued.

(i) Grades obtained in an extra or optional course shall be reflected in the transcripts.

(j) A student who fails an extra or optional course shall not be penalized as long as he/she has the minimum prescribed course units.

(k) Under exceptional circumstances, such as medical or compassionate grounds, supported by authentic written evidence, examinations may be held for the candidate.

A special examination shall be treated as a regular written examination.

7.2 GRADING SYSTEM

Percentage	Grade	Remarks
75 – 100	A	Distinction
65 – 74	B	Credit
50 – 64	C	Pass
Below 50	E	Fail

7.3 THESIS WRITING

(a) A student shall, during the degree programme, write a thesis on a specific topic in Mathematics.

(b) Thesis supervisor(s) shall be appointed for the students at the start of the second semester;

Where departmental rules are silent the common regulations for submission and examination of the Institute of Graduate Studies, Research and Extension (IGSRE) shall apply.

8. LEARNING AND TEACHING METHODS

A problem solving approach shall be used with emphasis on library research, internet, open problems, project, modeling and seminar.

9. <u>COURSE STRUCTURE</u>

YEAR ONE

SEMESTER I

(Core Courses)

WMA 5111C: Abstract Integration I	4
WMA 5113C: Functional Analysis I	4
WMA 5115C: General Topology I	4

Elective Courses (Optional)

WMA 5117E: Complex Analysis I	4
WMA 5121E: Group Theory	4

SEMESTER II

Core Courses

WMA 5112C: Abstract Integration II	4
WMA 5114C: Functional Analysis II	4
WMA 5190C: Research Methodology	6

Elective Courses (Choose any two)

WMA 5139E: Commutative Algebra	4
WMA 5119E: Differential Topology	4
WMA 5123E: Field Theory I	4
WMA 5125E: Algebraic Geometry	4

YEAR TWO

SEMESTER I

Core Courses

WMA 5190C: Proposal Writing	6

Electives (Optional)

WMA 5129E: Homological Algebra I	4
WMA 5131E: Operator Theory I	4
WMA 5133E: Banach Algebras I	4
WMA 5135E: Operators on Banach Spaces	4

SEMESTER II

Core Courses

WMA 519IC: Thesis Writing	9

Electives (Optional)

WMA 5137E: Topics in Operator Theory I	4
WMA 5141E: Non-Commutative Ring Theory	4
WMA 5143E: Coding Theory	4
WMA 5145E: Algebraic Topology I	4
TOTAL	**39**

9. COURSE DESCRIPTIONS

WMA 5111C: ABSTRACT INTEGRATION I

A survey of the Lebesque and borel measure on the real line. General measure theory in Abstract spaces. Measurable sets and measurable functions. Complex valued measurable functions. Complex valued measurable functions, functions between measurable spaces. Measurers and completion of an incomplete measure space. The Abstract Lebesque integral, Monotone Convergence theorem, Fatou's Lemma, which depend on a parameter. The Holder and Minkowski inequalities. The Lebesque spaces $L_p, 1 < p < \infty$, Essentially bounded functions. The space $L_{P.}$ The completeness theorem for $L_p (1 < P < \infty)$. Modes of convergence: Almost everywhere convergence, convergence in $L_{P.}$, almost uniform convergence, uniform convergence, convergence in measure, Riesz's theorem. Egeroff's theorem. Vitai's convergence theorem for $L_{P.}$

WMA 5113C: FUNCTIONAL ANALYSIS I

Completeness of a metric space. The contraction mapping theorem and its applications. The baire's category theorem and its applications. Compactness in a metric space. Relative compactness Stone-Weierstrass theorem. Normed linear spaces, banach space. Completion of a normed linear space. Strong topology in a normed linear space (n.l.s. for brevity). Subspaces and factor spaces. Riesz's lemma on "near orthogonality". Hamel and Schauder bases in n.l.s. finite dimensional n.i.s. Summability. Bounded linear transformations. Dual of a n.l.s. duals of some standard n.l.s. such as $L_p(1 \leq p < \infty)$. weak convergence and weak x-convergence in n.l.s. Banach Alaogolu's theorem.

WMA 5115C: GENERAL TOPOLOGY I

Prerequisites from set theory: Ordinals, cardinal numbers, Axiom of choice, partial order, order completeness. Zorn's lemma. Well-ordering theorem. General Cartesian products.

Topology: different sets of axioms, their equivalence. Topogization of sets. Continuity, homomorphism, open and closed maps. Weak topology. Identification topology, quotient maps. Identification spaces. Attaching of spaces.

Separation and countiability axioms, separability lindel of spaces: Regular normal, hereditarily normal, perfectly normal, completely normal spaces. Uryschn's characterization of normality. Tietze's characterization of normality covering; characterization of normality convergence: sequences, nets and filters. Adequacy of sequences in first countable spaces.

WMA 5117E: COMPLEX ANALYSIS I

Complex integraton, Cauchy's theorem and consequence. Laurent series. Calculus of residues. Inverse and implicit functions Rouch's theorem Harmonic and Subharmonic functions. The Poison integral. The mean-value property. Positive harmonic functions. Dirichlet's problem. The poisson-Jensen formula and related topics conformal mapping. Fractional linear transformations Normal families. The Riemann mapping theorem. Continuity at the boundary.

Conformal mapping of an annulus Maximum-Modulus principle. Schwarz's lemma. The phragmen-Lindel of and Hadamard's theorem. Entire functions with national values. Converse of the maximum modulus theorem.

WMA 5119E: DIFFERENTIAL TOPOLOGY I

Introduction of manifolds: Preliminaries on R and Euclidean spaces. Topological manifolds. Examples of manifolds, cutting and pasting. Abstract manifolds and their examples. Differential functions and mappings, rank of a mapping; Immersions, submanifolds, Lie groups, covering manifolds.

WMA 5121E: GROUP THEORY I

Group with operators, composition series, solvable groups, nilpotent groups.

Products direct and semi-direct groups, indecomposable groups. Free groups, free Abelian groups, Finitely generated Abelian groups.

WMA 5123E: FIELD THEORY I

Rings Integral domains, finite-dimensional lector space theory. Field theory including finite fields and Galois theory, Transcendental Extensions.

WMA 5125E: ALGEBRAIC GEOMETRY

Projective and affine varieties Isomorphism and birational isomorphism. The ring of regular functions and the field of rational functions on a variety. Dimension theory. Local properties, simple and singular points. Local ring at a point. The tangent space and its invariance. Differential mappings smooth varieties. Power series. Normal varieties. Divisors and differential forms, Dimension of a divisor. Elliptic curves. Genus of a curve. Abelian varieties.

WMA 5127: QUADRATIC FORMS

Quadratic forms over an arbitrary field matrix of a form.

The problem of equivalence. Bilinear forms and duality. Quadratic spaces and isometries. The orthogonal group. Isotrophy and normal bases.

Hyperbolic planes. The Wit group. Clifford algebras. Classification of quadratic form over the real and complex fields, and over finite field.

Introduction to p-adic numbers quadratic forms over a p-adic field. Norm residue symbol an the p-adic invariants. Quadratic forms over the rational field. The hasse-Minikowski theorem.

WMA 5129E: HOMOLOGICAL ALGEBRA I

Modules, diagrams and functions: modules, modules, diagrams, direct sums, free and projective modules, the functor Hom, categories, functors.

Homology of complexes: Differential groups, complexes, cohomology the exact homology sequences. Some diagram lemmas additive relations, singular homology, homology, axioms of homology.

Extensions and resolutions: extensions of modules, addition for extensions obstruction to the extension of a homomorphism. The universal coefficient theorem for cohomology, composition of extensions. Resolutions. Injective modules. Injective resolutions.

WMA 5131E: OPERATOR THEORY I

Calculus of projectors in Hilbert space. Convergence notions for sequences of projectors. Square roots of positive operators Partial Isometries Polar decomposition. Spectral theorems for compact normal and compact self-adjoint operators. Fredholm alternative in a Hilbert space. Applications to integral equations. Bounded integral operators in L^2 – spaces: Hilbert-Schmidt Operators and Carleman Operators. Matrix operators.

WMA 5133E: BANACH ALGEBRAS I

Normed algebras. Adjunction of the identity. The radical in a normal algebra. Banach algebras with identity. Resolvement in a Bananch algebra with identity. Regular representation of a normed algebra. Symmetric algebras. Positive functionals normed symmetric algebras positive functional in a symmetric banach algebra. Commutative normed algebras: Realization of a commutative normed algebra in the form of an algebra of functions. Homomorphism and normed algebra in the form of an algebra of functions. Homomorphism and isomorphism of commutative algebras. Shilev boundary. Regular algebras. Primary ideals.

WMA 5135E: OPERATORS ON BANACH SPACES

Topological vector spaces. Product spaces, subspaces. Direct sums, quotient spaces. Linear manifolds and pyperplains. Locally convex spaces: Reflexive spaces, topological spaces, separation theorem, equicontinuity. Reflexive spaces. Conjugate spaces. Convex positive linear forms and mappings, weak operators. Extreme points, extreme states, Krein-Millmann theorem. Banach algebras.

WMA 5137E: TOPICS IN OPERATOR THEORY I

Compact non-self-adjoint operators, Von-Neumann Schatten classes, Voltera operators. Triangular operators. Fredholm theory for trace-class operators. Superdiagonal representation of compact linear operators. Contractions, Dilations, Naimark’s theorem on dilations. Contractive semi groups, Dissipative operators.

WMA 5139: COMMUTATIVE ALGEBRA

Commutative rings and ideals. Prime and maximal ideal and their existence. Zero divisors and nilpotent-elements. The nil radical and Jacobson radical. Extensions and contractions of ideals. Modules and algebras, Submodules and quotient modules. Tensor products. Finitely generated modules and exact sequences. Rings and modules of fractions. Local rings and localization. Primary ideals and primary decomposition. Uniqueness theorems. Relation between primary ideals and prime powers.

WMA 5141E: NON COMMUTATIVE RING THEORY

Ring theory. Simple and semi-simple modules. Simple and semisimple Artinian rings, Radicals; Noetherian semi-prime rings.

WMA 5143E: CODING THEORY

Applications of algebra to algebraic coding theory: linear codes, cyclic codes, BH codes.

WMA 5145E: ALGEBRAIC TOPOLOGY I

Simple Homotopy theory: Mapping spaces, compact-open topology, H-spaces. Homotopy relations, homotopy equivalence. Fundametal group. Change of base points. Induced maps contractibility. Connectedness simple connectedness Retraction. Deformation. Some algebra: Push out, free group, free products, Seifert-ven Kampan's theorem.

WMA 5112C: ABSTRACT INTEGRATION II

Decomposition of measure: the Hahn decomposition theorem. The Jordan decomposition theorem, the Lebesque decomposition theorem. The Radon Nikodym theorem and the Riesz representation theorem for L_p-spaces.

Differentiation and absolute continuity. Vitali's covering theorem, Functions of bounded variation. Differentiation of point functions. Differential transformations.

Generation of measure: measures on algebras of sets, the extension of measures, outer measure, the Caratheodory and Hahn extension theorems. The Lebesque Stieltjes measures. The Riesz representation theorem for a bounded positive linear functional on C ([a,b]).

Product measures: Rectangles, the product measure theorem, sections, Monotone class Lemma, Tonnelli's theorem, Fubini's theorem Integration in locally compact topological spaces.

WMA 5114C: FUNCTIONAL ANALYSIS II

Uniform boundness principle, Banach-Steinhouse theorem and its applications. Closed linear transformations. The closed graph theorem and the open mapping theorem applications. Pre-Hilbert spaces. Cauchy-Schwarz inequality, Bessel's inequality, polarization identity. Basic geometry of Hilbert spaces. Orthonormal basis for a Hilbert spaces. Projection Theorem. Riesz's representation theorem for a bouded linear functional on a Hilbert space. Reflexity of Hilbert spaces. Adjoint of a alinear transformation in Hilbert spaces. The algebra B(X,Y) of bounded linear transformation from a n.l.s. X in to Y. Invertible linear transformations, Banach's theorem on inverse. Neumann's series. Transpose of an element in B(X,Y). spectrum of a linear operator –its components; point spectrum, continous spectrum, residual spectrum. Spectral radius, spectral mapping theorem. Approximate eigen-value and eigen-vector for an operator $T \in B(X)$, numerical range for a T in B(X), where X is a Hilbert space.

Compact operators in normed linear spaces and Hilbert spaces spectrum of a compact operator.

Elementary study of bounded self-adjoint, unitary and normal operators in a Hilbert space and some simple results about their spectrum.

WMA 5190C: RESEARCH METHODOLOGY

Types of research. Research Methods, and instruments. Identification of problems, problem statement, rationale, study objectives, literature review, budget, bibliography. Report writing: title, abstract, introduction, objectives, methods and materials, results, seminars.

WMA 5191C: PROPOSAL WRITING

Proposal writing and open problems.

WMA 5199C: THESIS

Research Methodology. Open problem solving. Thesis Writing.