# MASTERS OF SCIENCE IN APPLIED MATHEMATICS

## 1. INTRODUCTION

Applied mathematics is a branch of mathematics that concerns itself with mathematical methods that are typically used in science, engineering, business, and industry. Thus, "applied mathematics" is a mathematical science with specialized knowledge. The term "applied mathematics" also describes the professional specialty in which mathematicians work on practical problems; as a profession focused on practical problems, applied mathematics focuses on the formulation and study of mathematical models. In the past, practical applications have motivated the development of mathematical theories, which then became the subject of study in pure mathematics, where mathematics is developed primarily for its own sake. Thus, the activity of applied mathematics is vitally connected with research in pure mathematics.

This program gives students a broad background in mathematics, placing an emphasis on areas with the highest demand in applications: numerical methods and scientific computation, mathematical modeling, discrete mathematics, mathematical materials science, optimization and operations research. In addition to these advanced areas of specialization, students are encouraged to acquire breadth by choosing elective courses in other fields that complement their studies in applied mathematics. Students have a choice of completing their master’s thesis or project in cooperation with one of the department’s established industrial partners. The program provides a suitable foundation for the pursuit of a Ph.D. degree in applied mathematics or a related field, or for a career in industry immediately after graduation.

## 2. ADMISSION REQUIREMENTS

A bachelor’s degree is required for admission to all Masters programs. A basic knowledge of undergraduate analysis, linear algebra and differential equations is assumed for applicants to the master’s programs in applied mathematics and industrial mathematics. A strong background in mathematics, which should include courses in undergraduate analysis and linear algebra, is assumed for applicants to the master’s program in financial mathematics. Typically, an entering student in the Master of Science in applied statistics program will have an undergraduate major in the mathematical sciences, engineering or a physical science; however, individuals with other backgrounds will be considered. In any case, an applicant will need a strong background in mathematics, which should include courses in undergraduate analysis and probability. Students with serious deficiencies may be required to correct them on a noncredit basis.

Candidates for the master of mathematics for educators degree must have a bachelor's degree and must possess a background equivalent to at least a minor in mathematics, including calculus, linear algebra, and statistics. Students are encouraged to enroll in courses on an ad hoc basis without official program admission. However, (at most) four such courses may be taken prior to admission.

In addition to fulfilling the common university regulations, an applicant for the degree of Master of Science in Applied mathematics should satisfy the following requirements.

(a) Hold a Bachelor of Science degree at least at lower second class honours from a recognized university.
(b) Must have taken mathematics as a major subject in the first degree with an average of at least Grade B or its equivalent.

**3.DEGREE REQUIREMENTS**

The master's thesis is an original piece of mathematical research work which focuses on advancing the state of the mathematical art. The master's project consists of a creative application of mathematics to a real-world problem. It focuses on problem definition and solution using mathematical tools. The master's practicum requires a student to demonstrate the integration of advanced mathematical concepts and methods into professional practice. This could be done through an internship in industry or an applied research laboratory. The remaining courses may be chosen from the graduate offerings of the department of pure and applied mathematics

Upper-level undergraduate mathematics courses or a two-course graduate sequence in another department may be taken for graduate credit, subject to the approval of the departmental Graduate Committee. Candidates are required to successfully complete the graduate seminar.

## 4. OBJECTIVES

To produce graduates:

(i) With a wide background knowledge and basis techniques in Applied mathematics

(ii) Who are able to carry out research and undertake Ph.D. research in their respective areas of specialization.

(iii) With the ability to take up employment in public and private sectors of national and international spheres.

## 5. SUBMISSION AND PROCESSING OF APPLICATIONS FOR REGISTRATION

The common procedures approved by the senate shall apply.

## 6. DURATION OF THE PROGRAMME

The programme shall normally take a minimum of two (2) academic years and a maximum of four (4) years.

## 7. PROGRAMME REQUIREMENTS

(a) The program requires a minimum of thirty nine (39) units of coursework including thesis.

(b) A student may take extra courses over and above the required number of units.

(c) A student may also choose other courses from other department of BUC subject to the approval of the department.

## 8. EVALUATION

### 8.1 COURSEWORK AND EXAMINATION

(a) Each course shall be examined by a 4-hour end of semester written examination. This will account for 60% of the total mark in each course.

(b) Each course shall be examined by continuous coursework assessment comprising seminar papers, projects, reports, formal tests and participation in learning activities. This will account for 40% of the total mark in each course.

(c) The pass mark in each course (continuous assessment and written examination) shall be 50%.

(d) A candidate who fails in more than two courses shall be discontinued.

(e) A candidate who fails less in less than two course shall sit for a supplementary examination.

(f) Each supplementary examination shall be awarded a maximum of 50%.

(g) A candidate who fails any paper taken as a supplementary examination shall be discontinued.

(h) Grades obtained in an extra or optional course shall be reflected in the transcripts.

(i) A student who fails an extra or optional course shall not be penalized as long as he/she has the minimum prescribed course units.

(j) Under exceptional circumstances, such as medical or compassionate grounds, supported by authentic written evidence, examinations may be held for the candidate.

A special examination shall be treated as a regular written examination.

## 8.2 GRADING SYSTEM

| Percentage | Grade | Remarks |
|---|---|---|
| 75 – 100 | A | Distinction |
| 65 – 74 | B | Credit |
| 50 – 64 | C | Pass |
| Below 50 | E | Fail |

## 8.3 THESIS WRITING

(a) A student shall, during the degree programme, write a thesis on a specific topic in Mathematics.

(b) Thesis supervisor(s) shall be appointed for the students at the start of the second semester;

Where departmental rules are silent the common regulations for submission and examination of the Institute of Graduate Studies, Research and Extension (IGSRE) shall apply.

**8.4 THESIS WRITING**

(c) A student shall, during the degree programme, write a thesis on a specific topic in Mathematics.

(d) Thesis supervisor(s) shall be appointed for the students at the start of the second semester, through the active involvement of the student.

Where departmental rules are silent the common regulations for submission and examination of the School of Graduate Studies, (SGS) shall apply.

## 9. LEARNING AND TEACHING METHODS

A problem solving approach shall be used with emphasis on library research, open problems, project, modeling and seminar.

## 10. COURSE STRUCTURE

A student in applied in expected to take a minimum of four courses per semester.

| **First Year** | **No. of Units** |
|---|---|
| **SEMESTER I** | |
| **Core Course** | |
| WMA 5011 **:** Complex Analysis **I** | **4** |
| WMA 5021: Ordinary Differential Equations I | 4 |
| WMA 5031: Numerical Analysis I | 4 |
| WMA 5041: Partial Differential Equations I | 4 |
| | |
| **Elective(s) Course (to select any 3)** | |
| WMA 5051: Applied Dynamical systems I | 4 |
| WMA 5061: **Applied Functional Analysis I** | 4 |

| | |
|---|---|
| WMA 5071 Mathematics of Quantum Mechanics | 4 |
| WMA 5081: Fluid Mechanics I | 4 |
| WMA 5091: Operation Research I | 4 |
| WMA 5101: Lie Groups and Differential Equations I | 4 |

**SEMESTER II**

**Core Courses**

| | |
|---|---|
| WMA 5112: Applied Dynamical Systems II | 4 |
| WMA 5122: Ordinary Differential Equation II | 4 |
| WMA 5132: Numerical Analysis II | 4 |
| WMA 5142: Biomathematics I | 4 |
| WMA 5152:- Numerical Computational for Differential Equations | 4 |

**Elective(s) Courses (select any 1)**

| | |
|---|---|
| WMA 5162 Applied Dynamical Systems II | 4 |
| WMA 5172 Partial Differential Equation II | 4 |
| WMA 5182 Fluid Mechanics II | 4 |
| WMA 5192 Methods of Applied Mathematics II | 4 |
| WMA 5202 Finite Element Method I | 4 |
| WMA 5212 Methods in Fluid Mechanics II | 4 |
| WMA 5222 Distributions and Fourier analysis | 4 |
| WMA 5232: Numerical Analysis III | 4 |
| WMA 5242: Finite Element Method II | 4 |
| WMA 5252: Partial Differential Equation III | 4 |
| WMA 5262: Operation Research II | 4 |
| WMA 5272: Lie Groups and Differential Equations II | 4 |

**Second Year**

**<u>SEMESTER I</u>**

**Core Courses**

| | | | |
|---|---|---|---|
| WMA 5 890 | - | Thesis Writing | 6 |

**<u>SEMESTER II</u>**

**Core courses**

| | | | |
|---|---|---|---|
| WMA 5 893 | - | Thesis | 6 |

Each course is 4 units except WMA 5890 Thesis Research, which is equivalent to 6 units.

## COURSES DESCRIPTION

### WMA 5051: APPLIED DYNAMICAL SYSTEMS I

A basic introduction to the general theory of dynamical systems from a mathematical standpoint, this course studies the properties of continuous and discrete dynamical systems, in the form of ordinary differential and difference equations and iterated maps. Topics include contracting and expanding maps, interval and circle maps, toral flows, billiards, limit sets and recurrence, topological transitivity, bifurcation theory and chaos. Applications include classical mechanics and optics, inverse and implicit functions theorems, the existence and uniqueness of general ODEs, stable and center manifolds, and structural stability.

### WMA 5122: ORDINARY DIFFERENTIAL EQUATIONS II

Sturm-Liouvile systems and applications. Singular and regular self-adjoint boundary value problems for second and higher orders, and limit point cases. Asymptotic behavior of solutions of systems of Ordinary Differential Equations. Poicare-Bendixon Theory. Solutions of non linear Ordinary Differential Equations.

### WMA 5071 MATHEMATICS OF QUANTUM MECHANICS

The basis of quantum mechanics is the Schrodinger equation. The focus of this course will be on one dimensional Schrodinger equations. Topics include eigenvalue problems, bound states, scattering states, tunneling, uncertainty principle, dynmaics, semi-classical limit. The ideas will be illustrated through many examples.

### WMA 5061: APPLIED FUNCTIONAL ANALYSIS I

Metric spaces; Normed linear spaces; Banach spaces; Inner product spaces; Hilbert spaces; Fundamental theorems for normed linear spaces; Duality and reflexive spaces. Normed lilnear spaces, Banach spaces, compactness in a metric space.

Linear operators; Compact operators; Self-Adjoint operators; Spectra.

### WMA 5814 – APPLIED FUNCTIONAL ANALYSIS II

Spectral theory of bounded linear operators; Unbounded operators; Spectral theory of unbounded operators. Uniform boundness principle, Banach –stein house theorem, open mapping theorem, Reist's representation theorem.

**WMA 5855 – DISTRIBUTIONS AND FOURIER ANALYSIS**

Convergence. Generalized Functions (distributions); Differentiation, translations, multiplications and convolutions; Fourier transforMATHions; Tempered distributions.

**WMA 5081: FLUID MECHANICS I**

Gasdynamics and aerodynamics; basic laws and concepts governing compressive fluid flows, and conservation laws. Laws of thermodynamics. Ideal gas approxiMATHion. One-dimensional compressible and isentropic flows. Mach number and Mach cone. Subsonic, hypersonic and transonic flows. Flow through nozzles. Rankine-Hugoniot relations for normal and oblique shock waves. Solutions of two dimensional steady subsonic and supersonic flows. Prandtl-Meyer function. Hodograph and characteristic methods. Calculation of lift and drag for a thin aerofoil. Supersonic and hypersonic flows over a slender body. Magnetohohydrodynamics: basic equations hydromagnetic flows. Hartman and hydromagnetic coquette flow, flow through an annulus and unsteady flow over a flat plate.

**WMA 5101: LIE GROUPS AND DIFFERENTIAL EQUATIONS I**

Manifolds: submanifolds, transformation groups-Lie groups of transformations-One parameter Lie groups of transformations, infinitesimal transformations, vector fields , infinitesimal Generators. Lie algebras: Lie brackets. Symmetries of algebraic functions: invariant functions, infinitesimal invariance, local invariance, invariants and functional dependence, methods for constructing invariants. Groups and differential equations: invariance of differential equation, prolongation of vector fields.

**WMA 5823B: LIE GROUPS AND DIFFERENTIAL EQUATIONS II**

Lie Symmetries and Differential equations: prolongation-invariance of differential equation, prolongation of vector fields-extended group transformation; one dependent and one independent variable- one dependent and many independent variables. Calculation of Symmetry Groups. Group-Invariant solutions of differential equations. Symmetry solutions of partial differential equations. Prerequisites: Lie Groups And Differential Equations I

**WMA 5824 – FLUID MECHANICS II**

Element of wave motion and one-dimensional wave equations. Wave equation in two and three dimensions; linearization of equation; sound waves in fluid; diffraction of sound waves by boundaries and forMATHion of sound shadows; generation of sound general fluid motion; plane waves in tubes; cylindrical, spherical and water waves; equations of motion; plane waves in deep and shallow water; and liquid motion under gravity. One-dimensional wave on horizontal bed, serges in a uniform channel and flow over a small obstacle. Hydrodynamic equations of binary fluid and large scale ocean waves in the absence of tidal forces.

**WMA 5826 – FLUID MECHANICS III**

Nature of MHD (Magnetohydrodynamics). Maxwell's electromagnetic field equations for medium at rest and in motion. Equations of motion of electrical charge, fundamental equations of MHD, and their properties. Parameters in MHD, Alfuen's theorem and MHD flows such as flow between parallel plates in a pipe under external magnetic force near a stagnation point. Boundary of fluid of small and large electrical conductivity. MHD flows in one dimension: stability, equations, generalized Hogoniot relations, waves including Alfluen's waves, linearised equations, reflection and transmissions at discontinuity in dense, oblique and cylindrical shock waves. Two and three dimensional MHD flows.

**WMA 5828 – FLUID MECHANICS IV**

Fundamentals of radioactive transfer. Equations of transfer of radiation. Radiation gas dynamics: fundamental equations, boundary conditions, similarity parameters, waves and shock waves, and heat transfer.

**WMA 5031: NUMERICAL ANALYSIS I**

Interpolation and Approximation: Limitations on polymonial interpolation. The Fourier approxiMATHion. Interpolation in two variables in two variables. Rational approxiMATHion. Minimum – maximum error techniques. Constructing minimax approxiMATHions. The calculation of eigen-vectors of symmetric MATHrices. The eigen values and eigen vectors of symmetric MATHrices. Methods of non-symmetric MATHrices. The LR and QR logarithms. Errors in computed eigen-values and eigen-vectors. Deflation.

**WMA 5031: NUMERICAL ANALYSIS II**

Numerical integration: Singular integrals, integration over a multidimensional hypercube. Error determination. Unequal intervals. Methods based on higher derivatives. Extrapolation methods, stiff equations. Stability. Boundary value problems. Chebyshev methods. Error estiMATHion. Solution of partial differential equations. Hyperbolic equations. Parabolic and elliptic partial differential.

**WMA 5842 – NUMERICAL ANALYSIS III**

Simultaneous non-linear equations and optimization. Univariate and Fibonacci searches. The method of Newtton-Raphson errors on the roots. Simultaneous linear equations. Error analysis. Iterative refinement. Round off error in iterative methods. Acceleration of stationary iterative processes. MATHrix inversion. Over determined systems of linear equations. Solving these systems of linear algebraic equations through the use of residue arithmetic.

**WMA 5844 – NUMERICAL ANALYSIS IV**

Ordinary differential equations; stability, convergence and accuracy, Boundary value and eigen-Value problems. Partial differential equations. Elliptic boundary value problems and initial value problems. Iterative methods for solving linear systems. The use of high speed computers. Rayleigh’s quotient, effect of coefficient errors on roots and ill-conditioned polynomials.

**WMA 5041: PARTIAL DIFFERENTIAL EQUATIONS I**

Linear second order equations in two or more variables. Initial and boundary value problems for elliptic, hyperbolic and parabolic equation, separation of variables, eigen function expansions, Greens functions and other special functions, Fourier series, Uniqueness and existence of solutions.

**WMA 5834 – PARTIAL DIFFERENTIAL EQUATIONS II**

Mathematical theory of the parabolic and hyperbolic equations. Cauchy problems: the Cauchy-Koureieskii theorems, equations with independent variables, characteristics, the method of successive approximations, and the Riemann function. The fundamental solution in two independent variables. Cauchy's problem in space of higher dimensions. Fundamental solution and the classical solution in two variables.

**WMA 5838 – PARTIAL DIFFERENTIAL EQUATIONS III**

Elements of Hilbert and Sobolev spaces. Trace, Sobolev Lemma and imbedding. Compactness, Coercivitity and Regularity. Sobolev equations. Fundamental solutions of Elliptic equations: Weak and strong derivatives. Extended sobolev in equalities in R and bounded domains. Imbedding theorems classical and generalized solutions of Dirichlet problems.

Existence theory of solutions. A priori inequalities and regularity on the boundary. General boundary conditions. Fundamental solutions of cauchy problems. Differentiability and smoothness of solutions. Uniqueness and differentiability theorems in Hilbert space. Solution of the initial boundary value problems. Analyticity and Asymptotic behavior of solutions of parabolic and elliptic equations.

**WMA 5021: Ordinary Differential Equations I**

Linear equations of order n with constant and variable coefficients. Linear independence, existence and uniqueness of solutions, Picard-Linderl of Peano and caratheodony theorems, linear systems of differential equations with singularities of first and second kinds, stability analysis, perturbation method, its application to differential equations and eigen value problems.

**WMA 5846 – METHODS OF APPLIED MATHHEMATHICS I**

Applications of complex variables: Contour integration, calculus or residues, saddle point methods, evaluation of integrals, analytical continuation. Cartesian tensors; algebra and calculus. Integral transforms: complex form of Fourier series, Fourier transform, Laplace transform pairs. Partial Differential Equations: Separation of variables, integral transform methods, Weiner-Hopf techniques.

**WMA 5847 – METHODS OF APPLIED MATHHEMATHICS II**

Special functions: Advanced concepts in Legendre functions, Bessel functions, hypergeometric functions, associated Legendre functions and spherical harmonics. Integral Equations: Classifications, degenerate kernels, Newman and Fredholm series, Schimidt-Hilbert theory, miscellaneous devices for solution including the Weiner-Hopf technique. Calculus of variation. Euler-Lagrange equation form Hamilton's principal, eigen values, Green's functions and generalized functions.

**WMA 5851 – GEOMETRY OF MANIFOLDS I**

Diferential manifold, curvature tensor. Parallel displacement. Riemanian metric. Christoffel symbols. Curvature with respect to Christoffel symbols. Differential operators. Geodesics coordinates. Riemanian curvature. Conformal curvature tensor. Orthogonal basis canonical congruences. Gaussian and Riccian curvature.

**WMA 5852 – GEOMETRY OF MANIFOLDS II**

Existence of normals. Gauss's formulae: Tensor derivative of the normal vector. Liners of curvature. Generalized Mainardi-Codazzi and Gauss equations Hyperquadratics. Subspaces of an Euclidean space. Evollute of V immersed in E. Spherical representations of V.

**WMA 5853 – GEOMETRY OF MANIFOLDS III**

Review of theory of curves. Fundamental forms, Are-length and surface area, normal curvature. Principal curvatures and directions. Gaussian and mean curvature lines of curvature. Asymptotic lines. Gauss-weigarten equations. The compatibility equations and the theorem of Gauss. The fundamental Theorem of surfaces. Tensors. Applications of tensors to the equations of surface theory.

**WMA 5853 – GEOMETRY OF MANIFOLDS III**

Mapping of surfaces. Isometric mappings. Geodesic curvature, geodesic coordinates, geodesic polar co-ordinates, Arc of minimum length. Surfaces with constant Gaussian curvature, Gauss-Bonnet theorem. Fibre bundles and vector bundles. Application of Lie-groups and Lie derivatives.

**WMA 5011 : COMPLEX ANALYSIS I**

Complex integration, Cauchy's theorem and consequence. Laurent series. Calculus of residues. Inverses and implicit functions, rouche's theorem. Harmonic and subharmonic functions. The poison integral. The mean-value property. Positive harmonic functions. Dirichlet's problem. The Poisson-Jensen formula and related topics. Conformal mapping. Fractional linear transforMATHions. Normal families. The Reimann mapping theorem.

Continuity at the boundary. Conformal mapping of an annulus maximum-modullus principle. Schwarz’s lemma. The phagemen-Lindel’s of an Hadamord’s theorem. Entire functions with rational values. Converse of the maximum modulus theorem.

**WMA 5822 – COMPLEX ANALYSIS II**

Entire and meromorphic functions. Zeros of holomorphic functions. Infinite products and partial fractions expansions. Weierstrass factorization theorem.Blaschke products. The Muntz-Szasz theorem. Elliptic functions. Global analytic functions, analytic continuation. Monodremy theorem. Complete analytic function. Elementary theory of Riemann surfaces.

**WMA 5820 – METHODS IN FLUID MECHANICS I**

Perturbation methods: Asymptolic expansion. Differential equations with an irregular singularity. Singularity. Singular perturbations. Limit processes, MATHching principles. Two-variable expansion procedures. Applications to non-linear oscillation, viscous flow, geophysics etc. numerical methods, simultaneous linear equations, Gaussian elimination, LV decomposition and sparse MATHrices, large linear systems.

**WMA 5848 – METHODS IN FLUID MECHANICS II**

Jacobi, Gauss-Seidel and relaxation methods, error analysis. Iterative solution of simultaneous equations. Partial differential equations, initial and boundary value problems. Numerical solution of problem involving Euler and Navier-Stokes equations, relation between time accurate and relaxation techniques, higher order implicit methods, accuracy, stability complexity. Application to selected problems.

**WMA 5854 – FINITE ELEMENT METHODS I**

Calculus of variations. Variatinal methods. Weighted residual methods. Construction of the piecewise Langrange and Hermite interpolation polynomials over the finite element in one, two and three space-dimensions. Integration over finite elements. Finite element methods. Stability and convergence of finite element methods.

**WMA 5854 – FINITE ELEMENT METHODS II**

Solutions of initial value problems and boundary value problem in linear and non-linear differential equations. Application to solve parabolic, elliptic and hyperbolic partial differential equations. Initial value and boundary value problems. Finite element methods to solve the two and three dimensional flow problems in Fluid mechanics. Finite element methods in Computer-aided design.

**WMA 5091: OPERATION RESEARCH I**

Introduction technology and classification of games, Definition of rectangular Games, Rectangular Games with saddle-points. Mixed strategies, Geometrical background, proof of the fundamental theorem of arbitrary rectangular Games. The set of solutions, some properties of MATHrices, Determination of all solutions. Methods of approxiMATHing the value of a Game. Solutions of (MXN) Games by linear programming. Game theory and Markov processes.

**WMA 5850 – OPERATION RESEARCH II**

Unconstrained non-linear algorithms: Direct search method, Gradient method. Constrained non-linear algorithms: separable, quadratic and geometric programming. Queuing models, K-station series models. Analysis of queues by imbedded Markov chains: M/G/IEM/M/M/1, M/EM/1, M/D/1., D/M/1 Queuing models.

**WMA 5891 – RESEARCH METHODOLOGY**

The purpose of research, selecting a research topic and designing a research project proposal. Discussion of some proposals, employing various methods; literature research; research writing and presentation. Seminar are presented and discussed by both students and teaching members of the department.

**WMA 5892 – PROPOSAL WRITING**

Proposal writing.

**WMA 5893 – THESIS**

Thesis writing.